

# Kaon HBT radii from perfect fluid dynamics using the Buda-Lund model

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In this paper we summarize the ellipsoidally symmetric Buda-Lund model's results on HBT radii. We calculate the Bose-Einstein correlation function from the model and derive formulas for the transverse momentum dependence of the correlation radii in the Bertsch-Pratt system of out, side and longitudinal directions. We show a comparison to  $\sqrt{s_{NN}} = 200\text{GeV}$  RHIC PHENIX two-pion correlation data and make prediction on the same observable for different particles.

## 1. Perfect fluid hydrodynamics

Perfect fluid hydrodynamics is based on local conservation of entropy and four-momentum. The fluid is perfect if the four-momentum tensor is diagonal in the local rest frame. The conservation equations are closed by the equation of state, which gives the relationship between energy density  $\epsilon$ , pressure  $p$ . Typically  $\epsilon - B = \kappa(p + B)$ , where  $B$  stands for a bag constant ( $B = 0$  in the hadronic phase, non-zero in a QGP phase), and  $\kappa$  may be a constant, but can be an arbitrary temperature dependent function.

There are only a few exact solutions for these equations. One (and historically the first) is the famous Landau-Khalatnikov solution discovered more than 50 years ago [1, 2, 3]. This is a 1+1 dimensional solution, and has realistic properties: it describes a 1+1 dimensional expansion, does not lack acceleration and predicts an approximately Gaussian rapidity distribution.

Another renowned solution of relativistic hydrodynamics is the Hwa-Bjorken solution [4, 5, 6], which is a simple, explicit and exact, but accelerationless solution. This solution is boost-invariant in its original form, but

this approximation fails to describe the data [7, 8]. However, the solution allowed Bjorken to obtain a simple estimate of the initial energy density reached in high energy reactions from final state hadronic observables.

There are solutions which interpolate between the above two solutions [9, 10], are explicit and describe a relativistic acceleration.

## 2. The Buda-Lund model

We focus here on the analytic approach in exploring the consequences of the presence of such perfect fluids in high energy heavy ion experiments in Au+Au collisions at RHIC. Such exact analytic solutions were published recently in refs. [9, 10, 11, 12, 13]. A tool, that is based on the above listed exact, dynamical hydro solutions, is the Buda-Lund hydro model of refs. [14, 15].

The Buda-Lund hydro model successfully describes BRAHMS, PHENIX, PHOBOS and STAR data on identified single particle spectra and the transverse mass dependent Bose-Einstein or HBT radii as well as the pseudorapidity distribution of charged particles in central Au+Au collisions both at  $\sqrt{s_{NN}} = 130$  GeV [16] and at  $\sqrt{s_{NN}} = 200$  GeV [17] and in p+p collisions at  $\sqrt{s} = 200$  GeV [18], as well as data from Pb+Pb collisions at CERN SPS [19] and h+p reactions at CERN SPS [20, 21]. The model is defined with the help of its emission function; to take into account the effects of long-lived resonances, it utilizes the core-halo model [22]. It describes an expanding fireball of ellipsoidal symmetry (with the time-dependent principal axes of the ellipsoid being  $X$ ,  $Y$  and  $Z$ ).

## 3. HBT from the Buda-Lund model

Let us calculate the two-particle Bose-Einstein correlation function from the Buda-Lund source function of the Buda-Lund model as a function of  $q = p_1 - p_2$ , the four-momentum difference of the two particles. The result is

$$C(q) = 1 + \lambda e^{-q_0^2 \Delta\tau_*^2 - q_x^2 R_{*,x}^2 - q_y^2 R_{*,y}^2 - q_z^2 R_{*,z}^2}. \quad (1)$$

with  $\lambda$  being the intercept parameter (square of the ratio of particles emitted from the core versus from the halo [22]), and

$$\frac{1}{\Delta\tau_*^2} = \frac{1}{\Delta\tau^2} + \frac{m_t}{T_0} \frac{d^2}{\tau_0^2}, \quad (2)$$

$$R_{*,x}^2 = X^2 \left( 1 + m_t (a^2 + \dot{X}^2) / T_0 \right)^{-1}, \quad (3)$$

$$R_{*,y}^2 = Y^2 \left( 1 + m_t (a^2 + \dot{Y}^2) / T_0 \right)^{-1}, \quad (4)$$

$$R_{*,z}^2 = Z^2 \left( 1 + m_t(a^2 + \dot{Z}^2)/T_0 \right)^{-1}, \quad (5)$$

with  $\dot{X}, \dot{Y}, \dot{Z}$  being the time-derivative of the principal axes,  $m_t$  the average transverse mass of the pair.  $T_0$  is the central temperature at the freeze-out,  $\Delta\tau$  is the mean emission duration and  $\tau_0$  is the freeze-out time. Furthermore,  $a$  and  $d$  are the spatial and time-like temperature gradients, defined as  $a^2 = \left\langle \frac{\Delta T}{T} \right\rangle_\perp^2$  and  $d^2 = \left\langle \frac{\Delta T}{T} \right\rangle_\tau^2$ . From the mass-shell constraint one finds  $q_0 = \beta_x q_x + \beta_y q_y + \beta_z q_z$ , if expressed by the average velocity  $\beta$ . Thus we can rewrite eq. 1 with modified radii to

$$C(q) = 1 + \lambda_* \exp \left( - \sum_{i,j=x,y,z} R_{i,j}^2 q_i q_j \right), \text{ where} \quad (6)$$

$$R_{i,i}^2 = R_{*,i}^2 + \beta_i^2 \Delta\tau_*^2, \text{ and } R_{i,j}^2 = \beta_i \beta_j \Delta\tau_*^2, \quad (7)$$

From this, we can calculate the radii in the Bertsch-Pratt frame [23] of out ( $o$ , pointing towards the average momentum of the actual pair, rotated from  $x$  by an azimuthal angle  $\varphi$ ), longitudinal ( $l$ , pointing towards the beam direction) directions and side ( $s$ , perpendicular to both  $l$  and  $o$ ) directions. The detailed calculations are described in ref. [24]. These include azimuthally sensitive oscillating cross-terms. However, due to space limitations, the angle dependent radii are not shown here. If one averages on the azimuthal angle, and goes into the LCMS frame (where  $\beta_l = \beta_s = 0$ ), the Bertsch-Pratt radii are:

$$R_o^2 = (R_{*,x}^{-2} + R_{*,y}^{-2})^{-1} + \beta_o^2 \Delta\tau_*^2, \quad (8)$$

$$R_s^2 = (R_{*,x}^{-2} + R_{*,y}^{-2})^{-1}, \quad (9)$$

$$R_l^2 = R_{*,z}^2. \quad (10)$$

These can be fitted then to the data [25] as in ref. [26], see fig. 1. This allows us to predict the transverse momentum dependence of the HBT radii of two-kaon correlations as well: if they are plotted versus  $m_t$ , the data of all particles fall on the same curve. This is also shown for kaons on fig. 1.

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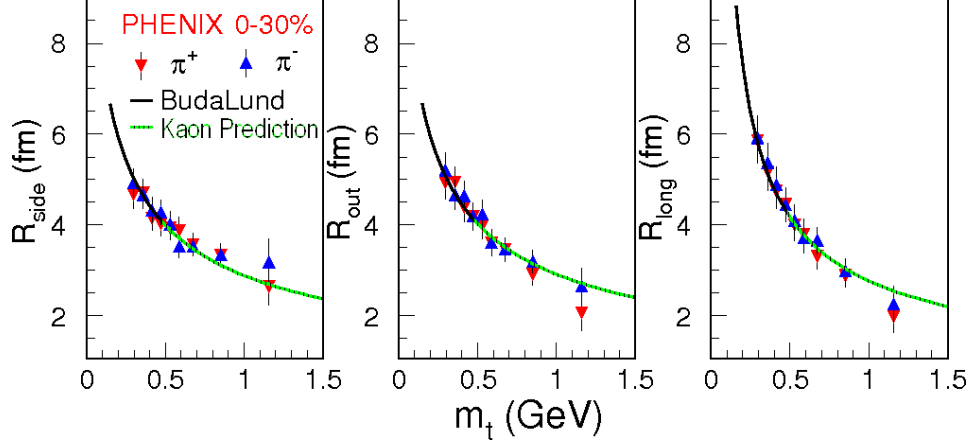


Fig. 1. HBT radii from the axially Buda-Lund model from ref. [26], compared to data of ref [25]. We also show a prediction for kaon HBT radii on this plot: these overlap with that of pions if plotted versus transverse mass  $m_t$ .

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